IMPORTANT QUESTIONS – 2019-20
CLASS – XII – MATHEMATICS

1. **Prove that :** $\tan^{-1}\left[\dfrac{3a^2x - x^3}{a(a^2 - 3x^2)}\right] = 3\tan^{-1}\left(\dfrac{x}{a}\right)$

2. **Prove :** $\sin^{-1}\left(\dfrac{x+\sqrt{1-x^2}}{\sqrt{2}}\right) = \dfrac{\pi}{4} + \sin^{-1}x, -1 \le x \le 1.$

3. **Write simplest form** $\cos^{-1}\left(\dfrac{3}{5}\cos x + \dfrac{4}{5}\sin x\right)$

4. If $\cos^{-1} a + \cos^{-1} b + \cos^{-1} c = \pi$, prove that $a^2 + b^2 + c^2 + 2abc = 1$.

5. $\sin^{-1}(x^2\sqrt{1-x^2} + x\sqrt{1-x^4})$ **,** write simplest form.

6. **tan** $(\cos^{-1}x) = \sin\left(\cot^{-1}\dfrac{1}{2}\right)$, Solve for x.

7. $\sin^{-1}\dfrac{3x}{5} + \sin^{-1}\dfrac{4x}{5} = \sin^{-1}x$, Solve for x.

8. **Given** $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ verify that BA= 6I, use the result to solve the system x – y = 3, 2x + 3y + 4z=17, y + 2z = 7.

9. 10 Students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises Hard Workers. And the second group has honest and law abiding students and the third group contains Vigilant and Obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of first and second group is four times that of the third group. Using matrix method, find the number of students in each group. Apart from the values , Hard work, Honesty and Respect for law, Vigilance and Obedience, Suggest one more value, which in your opinion , the school should consider for awards.

10. $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3.$

11. $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$, where p is any scalar.

12. $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

13. Two schools A and B want to award their selected students on the values of Sincerity, Truthfulness and helpfulness. the school A wants to award Rs. x each Rs. y each and Rs. z each for the three respective values to 3, 2 and 1 students respectively with a total award money of Rs. 1,600. School B wants to spend Rs. 2300 to award

its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of ward for one prize on each values is Rs. 900. Using matrices find the award money for each value. Apart from these three values, suggest one more value which should be considered for award.

14. Area of a triangle with vertices (k, 0) , (1, 1) and (0, 3) is 5 sq units. Find the value (s) of k.

15. If $X_{m\times 3}Y_{p\times 4} = Z_{2\times b}$, find the values of m, p and b.

16. Using elementary transformations, find the inverse of the matrix $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{vmatrix}$

17. Express the matrix $\begin{vmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{vmatrix}$ as the sum of symmetric and a skew symmetric matrix.

18. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = x^2 - 2x - 3$, show that f(A)=0

19. Find the matrix X such that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.

20. Using properties of determinants, prove that:

$$\begin{bmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{bmatrix} = (ab+bc+ca)^3.$$

21. Using properties of 3determinants, prove that $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = (x+y+z)(x-z)^2$.

22. If for real numbers a,b and c $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 0$, then. show that a+b+c+ = 0 or a = b= c.

23. Using properties of determinants, prove that $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+2b & a+b & a \end{vmatrix} = 9b^2(a+b)$.

24. Using matrix metho1d, solve the following system of equations for x, y and z:

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10, \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10, \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13.$$

25. Find $\frac{dy}{dx}$, if $y = \cos^{-1}\left(\frac{3x + 4\sqrt{1-x^2}}{13}\right)$.

26. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, when $x \neq 0$.

27. If $e^x + e^y = e^{x+y}$, prove that $\frac{dy}{dx} + e^{y-x} = 0$

28. If $y = \left(x + \sqrt{1+x^2}\right)^n$, then show that $(1+x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = n^2y$.

29. Find $\dfrac{dy}{dx}$, if $y = (\cos x)^x + (\sin x)^{\frac{1}{x}}$.

30. If x = a(cos t + t sin t) and y = a(sin t – t cos t), $0 < t < \dfrac{\pi}{2}$, find $\dfrac{d^2x}{dt^2}, \dfrac{d^2y}{dt^2}$ *and* $\dfrac{d^2y}{dx^2}$.

31. If y = 3cos(log x) + 4 sin(log x), show that $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

32. If cos y = x cos (a + y) with cos a ≠ ±1, prove that $\dfrac{dy}{dx} = \dfrac{\cos^2(a+y)}{\sin a}$.

33. If x = a(cos 2t + 2t sin 2t) and y = a(sin 2t – 2t cos 2t), then find $\dfrac{d^2y}{dx^2}$.

34. $y = \sin^{-1}\left(\dfrac{\sqrt{1+x} - \sqrt{1-x}}{2}\right)$, find $\dfrac{dy}{dx}$.

35. y=log (x^x + $\operatorname{cosec}^2 x$), find $\dfrac{dy}{dx}$.

36. $y = x^{\cos x} + (\cos x)^{\sin x}$, find $\dfrac{dy}{dx}$.

37. An open box with a square base is to be made of a given quantity of metal sheet of area c^2. Show that maximum volume of box is $\dfrac{c^3}{6\sqrt{3}}$ cubic units.

38. Show that volume of greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle 30^0 is $\dfrac{4}{81}\pi h^3$.

39. Find the value p for which the curves $x^2 = 9p(9 - y)$ and $x^2 = p(y + 1)$ cut each other at right angles.

40. Find the equations of the normal lines to the curve $y = 4x^3 - 3x + 5$ which hare parallel to the line 9y + x + 3 = 0.

41. Find the equations of the normal to the curve $3x^2 - y^2 = 8$ parallel to the line x + 3y = 4.

42. Show that the curves $2x = y^2$ and 2xy = k cut at right angles, if $k^2 = 8$.

43. Find the approximate value of f(5.001, where $f(x) - x^3 - 7x^2 + 15$.

44. $\sqrt[3]{0.028}$.

45. Find the maximum area of an isosceles triangles inscribed in the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ with its vertex at one end of the major axis.

46. Find the interval in which the function $f(x) = \dfrac{4\sin x - 2x - x\cos x}{2 + \cos x}$, is strictly increasing or strictly decreasing in $(0, 2\pi)$.

47. If $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3 - y^3)$ show that $\dfrac{dy}{dx} = \dfrac{x^2\sqrt{1-y^6}}{y^2\sqrt{1-x^6}}$.

48. Show $\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2\,xyz\,(x+y+z)^3$

49. Find the inverse of the given matrix by using elementary row transformations.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

50. A school wants to award its students for the values of honesty, regulatory and hard work the total cash of Rs. 6,000. Three times the award money for hard work added to given for honesty amounts to Rs. 11,000. The award money given for honesty and hard work together is double the one given for regularity represent the above situation algebraically and find the award money for each value, using matrix method.